Closing Wed: HW_6A,6B,6C (7.3, 7.4, 7.5)
Closing next *Wed*: HW_7A,7B,7C (7.5, 7.7, 7.8)
Note: Exam 2 is *next* Thursday (Feb. 22nd)
Covers 6.4, 6.5, 7.1-7.5, 7.7, 7.8

Entry Task: Evaluate

$$\int \frac{x^2 + 6}{x^2 + 3x + 2}\,dx$$

Example: Evaluate

$$\int \frac{x^2 - x + 6}{x^3 + 3x}\,dx$$

Example:

$$\int \frac{x}{x^2 + 4x + 5}\,dx$$

How to integrate

A. Look for simplifications/substitutions
B. Products/Logs/Inverse Trig → BY PARTS
   Sin/Cos/Tan/Sec combos → TRIG
   Quadratic (under a radical) → TRIG SUB
   Rational Function → PART. FRAC.
C. If nothing seems to work, substitution.
   (u = inside, u = $\sqrt{}$, u = trig, u = e^x)

Examples:

1. $\int e^{\sqrt{x}} dx$

2. $\int \frac{\cos(x)}{4 - \sin^2(x)} dx$

3. $\int \frac{3}{x - 2\sqrt{x}} dx$

4. $\int e^{x} \cos(e^{x}) \sin^{3}(e^{x})\, dx$

How would you *start* these?

1. $\int \tan^3(x)\sec(x)\,dx$

2. $\int x^2 \ln(x)\,dx$

3 $\int x\sqrt{5-x^2}dx$

4. $\int \frac{\sqrt{x^2-1}}{x^2}dx$

5. $\int \frac{x^2+1}{x^2-2x-3}dx$

6. $\int x\tan^{-1}(x)dx$

7. $\int \frac{dx}{\sqrt{4x^2+8x-12}}dx$

7.7 Approximating Integrals (*Preview*)

Despite our best efforts in 7.1-7.5**,** <u>the vast majority of integrals CANNOT be done with any of our methods</u>.
So we usually have to approximate!

In this section we add two more approximation methods that are slightly more accurate. We already know left, right, and midpoint methods (but I included them below for completeness).

To approximate $\int_a^b f(x)dx$

1. Pick **n = number of subdivisions**.
   Compute $\Delta x = \frac{b-a}{n}$.
2. Label the tick marks: $x_i = a + i\Delta x$
3. Use an approximation method:

$$L_n = \Delta x[f(x_0) + f(x_1) + \cdots + f(x_{n-1})] \quad \text{(Left endpoint)}$$
$$R_n = \Delta x[f(x_1) + f(x_2) + \cdots + f(x_n)] \quad \text{(Right endpoint)}$$
$$M_n = \Delta x[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)] \quad \text{(Midpoint)}$$

New - Trapezoid Rule: (all the "middle terms" are multiplied by 2)

$$T_n = \frac{1}{2}\Delta x[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

New - Simpson's Rule: *n* must be even! (Alternating multiplying middle terms by 4 and 2)

$$S_n = \frac{1}{3}\Delta x[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

Example: (Note: none of our methods can integrate this)
Estimate

$$\int_0^3 \sqrt{100 - x^3}\,dx$$

Here are how to use the left, right, midpoint and trapezoid rules with n = 3 subdivisions:

$$L_3 = (1)\left[\sqrt{100-(0)^3} + \sqrt{100-(1)^3} + \sqrt{100-(2)^3}\right] \approx 29.5415$$

$$R_3 = (1)\left[\sqrt{100-(1)^3} + \sqrt{100-(2)^3} + \sqrt{100-(3)^3}\right] \approx 28.0855$$

$$M_3 = (1)\left[\sqrt{100-(0.5)^3} + \sqrt{100-(1.5)^3} + \sqrt{100-(2.5)^3}\right] \approx 29.0091$$

$$T_3 = \frac{1}{2}(1)\left[\sqrt{100-(0)^3} + 2\sqrt{100-(1)^3} + 2\sqrt{100-(2)^3} + \sqrt{100-(3)^3}\right] \approx 28.8135$$

Here is how to use Simpson's rule with n = 6 subdivisions (n has to be even to use this method):

$$S_6 = \frac{1}{3}\cdot\frac{1}{2}\left[\sqrt{100-(0)^3} + 4\sqrt{100-(0.5)^3} + 2\sqrt{100-(1)^3} + 4\sqrt{100-(1.5)^3}\right.$$
$$\left. + 2\sqrt{100-(2)^3} + 4\sqrt{100-(2.5)^3} + \sqrt{100-(3)^3}\right] \approx 28.9441$$

"Actual" Value (to 8 places after the decimal): 28.94418784